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I - Basic Theory

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Three-Dimensional Relativistic Field-Electron Interaction in a Multicavity High-Power Klystron

I - Basic Theory

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I

SUMMARY

This report describes a theoretical investigation of three-dimensional relativistic klystron interaction. The relativistic axisymmetric equations of motion are derived from the time-dependent Lagrangian function for a charged particle in an electromagnetic field. An analytical expression of the fringing RF electric and magnetic fields within and in the vicinity of the interaction gap and the space-charge forces between axially and radially elastic deformable rings of charges are both included in the formulation. This makes an accurate computation of electron motion through the tunnel of the cavities and the drift tube spaces possible. The method of analysis is based on a Lagrangian formulation. Bunching is computed by using a disk model of an electron stream in which the electron stream is divided into axisymmetric disks of equal charge and each disk is assumed to consist of a number of concentric rings of equal charges. The individual representative groups of electrons are followed through the interaction gaps and drift tube spaces.

INTRODUCTION

The klystron is one of the most versatile electron devices used for amplification and generation of energy at microwave frequencies at high power levels. It has found many applications in communications, radar, microwave energy sources for particle accelerations, microwave heating, and industrial processing. A recent advance in the depressed collector design (ref. 1) has made the high-power klystron a feasible high-efficiency microwave power source which is specially suited for transmission of large microwave power from space.

Efforts to compute rigorously relativistic three-dimensional axisymmetric electron motion in the klystron have been pursued at the Lewis Reseach Center. The present investigation serves as a continuation of such efforts with the objective of establishing a complete theory of relativistic klystron interaction.

In this study, the Lagrangian formulation of a hydrodynamic beam model is used. The electron stream entering the interaction gap is subdivided into representative charge groups. The individual charge groups are followed through each interaction gap and drift tube space until the output interaction gap is reached. Thus electron overtaking and crossover are appropriately dealt with. The electron stream is divided into N axisymmetric disks of equal charge per rf period, and each disk is assumed to consist of R concentric rings of equal charge. The rings are elastic and deformable in the axial and radial directions, and the disks are assumed to be thin and the rings narrow. The velocity of each ring, its phase with re-

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spect to the cavity gap voltage, and its radius at a specified position in various interaction gaps and in various drift tube spaces are computed from three-dimensional large signal programs.

With the space-charge effects on the bunching process included, the debunching effects of the space-charge forces are appropriately dealt with; furthermore, with the radial motion considered, radial beam loading and the effects of a multidimensional charge motion on the field-electron interaction process are accurately formulated.

This investigation separates conveniently into three parts:

(1) Formulation of relativistic equations of motion in an electromagnetic field
(2) A study of field-electron interaction in the first and second cavity gaps
(3) A study of the large signal energy exchange process in the third and succeeding cavities up to and including the output cavity

FORMULATION OF RELATIVISTIC EQUATIONS OF MOTION

The relativistic equations of motion of a charged particle in a curvilinear coordinate system, such as the circular cylindrical system, can be safely derived from an invariant formulation of particle dynamics, such as the Hamiltonian or the Lagrangian. Mathematically these functions are equivalent, but the Lagrangian method is somewhat more direct. In what follows, the Lagrangian formulation is used.

The Lagrangian of a particle of rest mass m_0 and charge e in an electromagnetic field determined by the potentials ϕ and $\vec{A}$ is

$$L(t) = -m_0c^2\sqrt{1-\beta^2} - e\phi + e(\vec{u}\cdot\vec{A}) \tag{1}$$

where $\beta = u/c$, as usual in relativistic formulas. If q_i, $i = 1,2,3$, are the three coordinates of any generalized coordinate system, which define the position of the charged particle, then the Lagrangian dynamic equations of motion are

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0 \qquad i = 1,2,3 \tag{2}$$

where $\partial L/\partial\dot{q}_i = p_i$ are the canonical momentum components, which together form the total momentum vector $\vec{p}$ of the particle. In circular cylindrical coordinates,

$$\vec{u} = \dot{r}\vec{a}_r + r\dot{\varphi}\vec{a}_\varphi + \dot{z}\vec{a}_z$$

and

$$u^2 = \dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2$$

The field vectors in free space or in vacuum are given by the following relations:

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$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

By performing the indicated differentiations, we obtain

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r} + eA_r \tag{3a}$$

$$p_z = m\dot{z} + eA_z \tag{3b}$$

$$p_\varphi = mr^2\dot{\varphi} + erA_\varphi \tag{3c}$$

Of these equations, the first two, (3a) and (3b), are linear momenta, and the third, (3c), is an angular momentum. They are the sums of the relativistic mechanical momenta, differing from the ordinary (nonrelativistic) momenta only by the Fitzgerald factor which appears implicitly in the relativistic mass m, and of a momentum of electromagnetic origin. It is assumed that the acceleration momentum is not important in the formulation. The relativistic mass m of the particle at velocity u is given by

$$m = \frac{m_0}{\sqrt{1 - \beta^2}}$$

(see appendix A). Next, the derivatives $\partial L/\partial q_i$ are given by

$$\frac{\partial L}{\partial r} = mr\dot{\varphi}^2 - e\frac{\partial \Phi}{\partial r} + e\left[\dot{r}\frac{\partial A_r}{\partial r} + \dot{\varphi}\frac{\partial}{\partial r}(rA_\varphi) + \dot{z}\frac{\partial A_z}{\partial r}\right] \tag{4a}$$

$$\frac{\partial L}{\partial z} = -e\frac{\partial \Phi}{\partial z} + e\left(\dot{r}\frac{\partial A_r}{\partial z} + r\dot{\varphi}\frac{\partial A_\varphi}{\partial z} + \dot{z}\frac{\partial A_z}{\partial z}\right) \tag{4b}$$

$$\frac{\partial L}{\partial \varphi} = -e\frac{\partial \Phi}{\partial \varphi} + e\left(\dot{r}\frac{\partial A_\varphi}{\partial \varphi} + r\dot{b}\frac{\partial A_\varphi}{\partial \varphi} + \dot{z}\frac{\partial A_\varphi}{\partial \varphi}\right) \tag{4c}$$

In the case of axisymmetric field, the field is rotationally symmetrical, L is independent of the azimuthal angle φ, and we obtain from equation (4c)

$$\frac{\partial L}{\partial \varphi} = 0 \tag{4d}$$

The relativistic dynamic equation of motion of the charged particle can now be obtained by substituting equations (3) and (4) into equation (2) with the help of the following relations:

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$$\vec{E} = -\nabla\Phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\frac{d}{dt} = \vec{u} \cdot \nabla + \frac{\partial}{\partial t}$$

When the result is regrouped,

$$\frac{d}{dt}\left(m\dot{r} - mr\dot{\varphi}^2\right) = e\left(E_r + r\dot{\varphi}B_z - \dot{z}B_\varphi\right) \tag{5a}$$

$$\frac{d}{dt}\, m\dot{z} = e\left(E_z + \dot{r}B_\varphi - r\dot{\varphi}B_r\right) \tag{5b}$$

$$\frac{d}{dt}\left(mr^2\dot{\varphi} + erA_\varphi\right) = 0 \tag{5c}$$

We notice that expansion of the left side of both equations (5a) and (5b) will contain a term dm/dt; thus, when the energy conservation relation (eq. (B-4c) of appendix B) is used, equations (5a) and (5b) become ($n_0 = e/m_0$)

$$\frac{\ddot{r}}{\sqrt{1-\beta^2}} - \frac{r\dot{\varphi}^2}{\sqrt{1-\beta^2}} = -\,|n_0|\left[E_r\left(1 - \frac{\dot{r}^2}{c^2}\right) - \frac{\dot{r}\dot{z}}{c^2}E_z + \left(r\dot{\varphi}B_z - \dot{z}B_\varphi\right)\right] \tag{6a}$$

$$\frac{\ddot{z}}{\sqrt{1-\beta^2}} = -\,|n_0|\left[E_z\left(1 - \frac{\dot{z}^2}{c^2}\right) - \frac{\dot{r}\dot{z}}{c^2}E_r + \left(\dot{r}B_\varphi - r\dot{\varphi}B_r\right)\right] \tag{6b}$$

Consider first the azimuthal equation of motion, equation (5c); the momentum integral yields

$$P_\varphi = \text{constant}$$

or

$$m_2 r_2^2\dot{\varphi}_2 + er_2A_{\varphi 2} = m_1 r_1^2\dot{\varphi}_1 + er_1A_{\varphi 1} \tag{7}$$

If electrons have started from the cathode, where $r = r_1$, $B_z = B_0$, and $\dot{\varphi} = \dot{\varphi}_1 = 0$, the total amount of magnetic flux passing through a circle of radius r_1 in plane 1 (fig. 1) is found by integrating the product of the axial magnetic field B_z and the differential ring-shaped area $2\pi r\,dr$ from zero to r_1 as follows:

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$$\Psi_1 = \int_0^{r_1} 2\pi r B_z \, dr$$

$$= 2\pi \int_0^{r_1} r(\nabla \times \vec{A})_z \, dr$$

$$= 2\pi r_1 A_{\varphi 1}$$

where $B_z = (\nabla \times \vec{A})_z$ and in cylindrical coordinates is given by

$$B_z = \frac{1}{r} \frac{\partial}{\partial r} (rA_\varphi)$$

for $\partial/\partial\varphi = 0$. We can now interpret the term $r_1 A_{\varphi 1}$ as the magnetic flux enclosed within the radius r_1. Similarly the term $r_2 A_{\varphi 2}$ is to be interpreted as $1/2\pi$ times the magnetic flux passing through plane 2 enclosed within a radius r_2. Thus equation (7) can be rearranged to yield Busch's theorem

$$m_2 r_2^2 \dot{\varphi}_2 = \frac{e}{2\pi} (\Psi_1 - \Psi_2)$$

or

$$\dot{\varphi} = - \frac{|n_0|}{2\pi r^2} \sqrt{1 - \beta^2} \, (\Psi_0 - \Psi) \qquad (8)$$

where $\Psi_1 = \Psi_0$ is the cathode flux and equation (8) is now referred to any plane perpendicular to the axial direction.

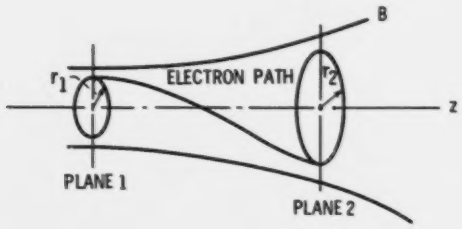

Figure 1. - Formulation of Busch's theorem.

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Equations (6a), (6b), and (8) form a system of three-dimensional relativistic equations of motion of an electron in an electromagnetic field, and they are rewritten as follows:

$$\ddot{r} = r\dot{\varphi}^2 - |\eta_0| \sqrt{1-\beta^2}\left[E_r\left(1 - \frac{\dot{r}^2}{c^2}\right) - \frac{\dot{r}\dot{z}}{c^2}E_z + r\dot{\varphi}B_z - \dot{z}B_\varphi\right] \tag{9a}$$

$$\ddot{z} = -|\eta_0| \sqrt{1-\beta^2}\left[E_z\left(1 - \frac{\dot{z}^2}{c^2}\right) - \frac{\dot{r}\dot{z}}{c^2}E_r + \dot{r}B_\varphi - r\dot{\varphi}B_r\right] \tag{9b}$$

$$\dot{\varphi} = -|\eta_0| \sqrt{1-\beta^2}\,\frac{\Psi_c - \Psi}{2\pi r^2} \tag{9c}$$

where the E's and B's are the electric and magnetic field components which may exist in both the interaction gaps and the drift spaces in a multicavity klystron. In an accurate treatment of klystron analysis, both the cavity circuit fields and the space-charge fields must be included in using equations (9). Thus

$$E_r = E_{r\text{-cct}} + E_{r\text{-sc}} \tag{10a}$$

$$E_z = E_{z\text{-cct}} + E_{z\text{-sc}} \tag{10b}$$

where $E_{r\text{-cct}}$ and $E_{z\text{-cct}}$ are the RF cavity gap fields at the beam position in the radial and axial directions, respectively, and $E_{r\text{-sc}}$ and $E_{z\text{-sc}}$ are the radial and axial space-charge fields, respectively.

Analytical expressions of the fringing RF electric and magnetic fields within and in the vicinity of the interaction gap obtained from a previous investigation (refs. 2 and 3) result in the following:

$$\left.\begin{aligned} E_r(r,z,t) &= E_0 F_r \exp(j\omega t) \\ E_z(r,z,t) &= E_0 F_z \exp(j\omega t) \end{aligned}\right\} \quad \text{for } -\ell \le z \le \ell \tag{11a, 11b}$$

and

$$\left.\begin{aligned} E_r(r,z,t) &= E_0 G_r \exp(j\omega t) \\ E_z(r,z,t) &= E_0 G_z \exp(j\omega t) \end{aligned}\right\} \quad \text{for } |z| \ge \ell \tag{12a, 12b}$$

where

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$$F_r = \sum_{n=1}^{\infty} \frac{J_1(\rho\lambda_n)}{J_1(\lambda_n)} \left(\frac{e^{m\ell}}{p_n - ma} + \frac{e^{-m\ell}}{p_n + ma} \right) \sinh\left(\frac{p_n z}{a}\right) e^{-p_n \ell/a}$$

$$- m \sinh(mz) \frac{J_1\left(r\sqrt{k^2 + m^2}\right)}{\left(\sqrt{k^2 + m^2}\right) J_0\left(a\sqrt{k^2 + m^2}\right)}$$

$$F_z = \cosh(mz) \frac{J_0\left(r\sqrt{k^2 + m^2}\right)}{J_0\left(a\sqrt{k^2 + m^2}\right)}$$

$$- \sum_{n=1}^{\infty} \frac{\lambda_n J_0(\rho\lambda_n)}{p_n J_1(\lambda_n)} \left(\frac{e^{m\ell}}{p_n - ma} + \frac{e^{-m\ell}}{p_n + ma} \right) e^{-p_n \ell/a} \cosh \frac{p_n z}{a}$$

$$G_r = \sum_{n=1}^{\infty} \frac{J_1(\rho\lambda_n)}{J_1(\lambda_n)} \left[\frac{\sinh(p_n + ma)\frac{\ell}{a}}{p_n + ma} + \frac{\sinh(p_n - ma)\frac{\ell}{a}}{p_n - ma} \frac{z e^{-(p_n z)/a}}{|z|} \right]$$

$$G_z = \sum_{n=1}^{\infty} \frac{\lambda_n J_0(\rho\lambda_n)}{p_n J_1(\lambda_n)} \left[\frac{\sinh(p_n + ma)\frac{\ell}{a}}{p_n + ma} + \frac{\sinh(p_n - ma)\frac{\ell}{a}}{p_n - ma} e^{-(p_n z)/a} \right]$$

in which E_0 is the electric field amplitude specified at the midplane of the cavity gap ($z = 0$ and $r = a$) and is related to the gap voltage by the following relations:

$$V_n = \int_0^{2\ell} E_0 \cosh m(z - \ell) dz = 2\left(\frac{E_0}{m}\right) \sinh(m\ell)$$

$$E_0 = \frac{mV_n}{2 \sinh(m\ell)} \qquad (13)$$

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where V_n is the gap voltage of the n^{th} cavity and m is the field-shape parameter. For a uniform field, $m = 0$.

The three-dimensional nonrelativistic space-charge fields have been derived (ref. 3) in terms of two static Green space-charge potentials $\mathcal{G}_\xi$ and $\mathcal{G}_\rho$. The three-dimensional relativistic space-charge fields may be readily obtained by relating these Green function potentials as static potentials in a moving coordinate system and by applying a proper Lorentz transformation. Furthermore, since in the moving frame of reference the vector potential function $\vec{A}'$ or $\vec{B}' = 0$, the fields in the laboratory frame can be found as follows:

$$E_{r-sc}(r,z,t;r',z') = -\frac{I_0}{NR\omega\varepsilon_0 a^2}\,\mathcal{G}'_\rho \tag{14a}$$

$$E_{z-sc}(r,z,t;r',z') = -\frac{I_0}{NR\omega\varepsilon_0 a^2}\,\mathcal{G}'_\xi \tag{14b}$$

$$E_{\varphi-sc}(r,z,t;r',z') = 0 \tag{14c}$$

$$B_{r-sc}(r,z,t;r',z') = 0 \tag{15a}$$

$$B_{z-sc}(r,z,t;r',z') = 0 \tag{15b}$$

$$B_{\varphi-sc}(r,z,t;r',z') = \left(\frac{u_z}{c^2}\right) E_{r-sc} \tag{15c}$$

where

$$\mathcal{G}'_\xi(\xi,\rho) = \sum_{n=1}^{\infty} \sum_{\text{all } \xi_0,\rho_0} \frac{J_0(\lambda_n\rho)J_0(\lambda_n\rho_0)}{J_1^2(\lambda_n)}\, e^{-\frac{\lambda_n}{\sqrt{1-\left(\frac{u_z}{c}\right)^2}}\,\text{sign}\,\frac{|\xi-\xi_0|}{\sqrt{1-\left(\frac{u_z}{c}\right)^2}}}$$

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$$\mathcal{G}'_{\rho}(\xi,\rho) = \frac{1}{\sqrt{1-\left(\frac{u_z}{c}\right)^2}} \sum_{n=1}^{\infty} \sum_{\text{all } \rho_0, \xi_0} \frac{J_1(\lambda_n \rho) J_0(\lambda_n \rho_0)}{J_1^2(\lambda_n)}$$

$$e^{-\frac{\lambda_n}{\sqrt{1-\left(\frac{u_z}{c}\right)^2}} |\xi - \xi_0|}$$

are the two modified Green function potentials. In these potential expressions, ξ and ρ are the two normalized coordinates at the field point, and ξ_0 and ρ_0 are the two normalized coordinates of the source points. The other symbols are defined as follows:

$I_0(\lambda_n)$, $I_1(\lambda_n)$	modified Bessel functions of zero and first order, respectively
$J_0(\lambda_n)$, $J_1(\lambda_n)$	Bessel functions of zero and first order, respectively
λ_n	n^{th} root of Bessel function $J_0(\lambda_n) = 0$
k	free space wave number, ω/c
p_n	$\sqrt{\lambda_n^2 - k^2 a^2}$
a	tunnel radius

When the expressions for the cavity gap fields and the relativistic space-charge fields (eqs. (10) to (15)) are substituted into equations (9a) and (9b) and the resultant equations are normalized with respect to the tunnel radius a, we obtain the relativistic three-dimensional equations of motion in dimensionless form as follows:

Normalized axial equation of motion:

$$\ddot{\xi} = \frac{\alpha P_0 \frac{m\ell}{\sinh(m\ell)}}{4 \frac{\ell}{a} \left(\beta_e a\right)^2} \cos\theta \left\{ (ka)^2 \ddot{\xi}_\mu \begin{pmatrix} G_\rho \\ F_\rho \end{pmatrix} + \left[(ka)^2 \dot{\xi}^2 - 1 \right] \begin{pmatrix} G_\xi \\ F_\xi \end{pmatrix} \right\}$$

$$+ \frac{\pi}{NR} \left(\frac{\omega_{p0}}{\omega}\right)^2 \frac{P_0}{\beta_e a} \left[1 - (ka)^2 \dot{\xi}^2\right] \mathcal{G}'_{\xi} + \frac{P_0^2}{2\rho} \left(\frac{\omega_{c0}}{\omega}\right)^2 \frac{B_r}{B_0} \frac{\Psi - \Psi_c}{\Psi_a} \qquad (16)$$

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Normalized radial equation of motion:

$$\ddot{\rho} = \frac{\alpha P_0 \frac{m\ell}{\sinh(m\ell)}}{4\frac{\ell}{a}\left(\beta_e a\right)^2}\cos\theta\left\{\left[1-(ka)^2\dot{\rho}^2\right]\begin{pmatrix}G_\rho\\F_\rho\end{pmatrix} - (ka)^2\ddot{\rho}\dot{\xi}\begin{pmatrix}G_\xi\\F_\xi\end{pmatrix}\right\}$$

$$+\frac{\pi}{NR}\left(\frac{\omega_{p0}}{\omega}\right)^2\frac{P_0}{\beta_e a}\left\{\left[1-(ka)^2(\dot{\rho}^2+\dot{\xi}^2)\right]\mathcal{G}'_\rho - (ka)^2\ddot{\rho}\dot{\xi}\mathcal{G}'_\xi\right\}$$

$$+\frac{P_0^2}{4\rho^3}\left(\frac{\omega_{p0}}{\omega}\right)^2\left\{\left(\frac{\Psi-\Psi_c}{\Psi_a}\right)^2 - 2\rho^2\frac{B_z}{B_0}\frac{\Psi-\Psi_c}{\Psi_a}\right\} \qquad (17)$$

$$P_0\sqrt{1-(u/c)^2} = \sqrt{\frac{1-(ka)^2(\dot{\rho}^2+\dot{\xi}^2)}{1+\frac{1}{4}(ka)^2\left(\frac{\omega_{c0}}{\omega}\right)^2\left(\frac{\Psi-\Psi_c}{\Psi_a}\right)^2}}$$

R_u relativistic velocity reduction factor, $\left(1+\frac{1}{2}\frac{V_0}{V_{eq}}\right)\left(1+\frac{V_0}{V_{eq}}\right)^{-1}$

r_c radius of cathode

u_0 relativistic dc beam velocity, $u_{00}R_u$

u_{00} nonrelativistic dc beam velocity, $\sqrt{2(e/m_0)V_0} = 5.93\times10^5\sqrt{V_0}$, m/sec

V_{eq} equivalent beam voltage, $m_0c^2/e = 5.11\times10^5$ volts

β_e ω/u_{00}

φ ωt

ξ normalized axial coordinate, z/a

$\dot{\xi}$ $d\xi/d\theta$

ρ normalized radial coordinate, ρ

$\dot{\rho}$ $d\rho/d\theta$

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$\Psi(\rho, \xi)$ $\quad 2\pi \int_0^{\rho} B_z(\rho, \xi)\rho \, d\rho$

Ψ_a $\quad \pi a^2 B_0$

Ψ_c $\quad$ cathode flux, $\pi a^2 B_c$

ω_{c0} $\quad$ nonrelativistic cyclotron angular frequency, $(e/m_0)B_0$

ω_{p0} $\quad$ nonrelativistic plasma angular frequency, $\sqrt{\frac{(e/m_0)I_0}{\varepsilon_0 \pi u_{00}}}$

We note that equations (16) and (17) are highly nonlinear, cannot be solved analytically, and must, therefore, be solved on a high-speed digital computer. Furthermore the modulation index α should be specified before equations (16) and (17) can be used.

For the input cavity gap (the first) and the second cavity gap α is, by definition, small (e.g., <0.02 and <0.07, respectively). In particular, for the first cavity, α_1 can be related to the power input to the cavity P_{in} and to the total shunt resistance of the cavity R_{sh} by the relation

$$\alpha_1 = \frac{V_1}{V_0} = \sqrt{\frac{2 R_{sh} P_{in}}{V_0}}$$

where $R_{sh} = R_{cav} + R_b + R_{ex}$; R_{cav} is the shunt resistance component due to cavity losses; R_b is the component from the beam loading, which may be negative but is ordinarily positive and often the dominant component of the total shunt resistance; and R_{ex} is the reflex load resistance due to external losses (load) coupled to the cavity (zero in the case of the input cavity). Generally R_{sh} is related to the unloaded Q_u and the R/Q value of the cavity by the relation

$$Q_u = \frac{R_{sh}}{R/Q}$$

Hence the value of R_{sh} can be obtained by values of Q_u and R/Q which are usually the design parameters for a given cavity.

CALCULATION OF BUNCHING CURRENT AND VELOCITY MODULATION IN SECOND CAVITY

Interaction of an unmodulated electron stream with the input cavity (first cavity) gap fields gives rise to the velocity modulation of the stream. As the beam moves along the first drift tube, the velocity variation of the stream is converted into density modulation, and the electron current at any point in the drift tube space can be computed from the kinematic of the electrons.

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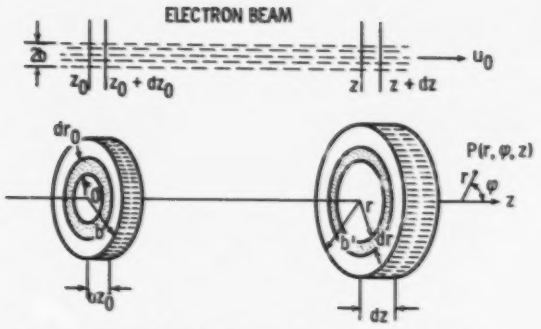

Figure 2. - Disk-ring model for computing induced current.

If we divide the beam into a number N of disks of electrons and we divide each disk further into R elementary charge rings (fig. 2), an elementary charge ring $2\pi r_0\, dr_0\, dz_0$ containing a charge density $\rho_0(z_0,r_0,t_0)$ at time t_0 at a later time t becomes $\rho(z,r,t)$ in an element of charge ring $2\pi r\, dr\, dz$. From the charge conservation principle, we relate these charges by the following equation:

$$\rho(z,r,t)2\pi r\, dr\, dz = \rho_0(z_0,r_0,t_0) \sum_{NR} |2\pi r_0\, dr_0\, dz_0|$$

The modulus is taken, since the sign of the charge is unchanged even if the electrons overtake one another. The summation over $(2\pi r_0\, dr_0\, dz_0)$ implies that all elements of the charge ring which entered the first cavity gap appear within the charge ring $2\pi r\, dr\, dz$ at the later time t as a result of overtaking and trajectory crossing. Thus

$$\rho(z,r,t) = \rho_0(z_0,r_0,t_0) \sum_{NR} \left| \frac{r_0\, dr_0\, dz_0}{r\, dr\, dz} \right| \tag{18}$$

where the charge density $\rho_0(z_0,r_0,t_0)$ is assumed constant over the beam cross section at the injection plane $z = z_0$, and, in terms of the beam current I_0,

$$\rho_0(z_0,r_0,t_0) = \frac{I_0}{\pi b^2 u_0}$$

We note that r_0 is the mean radius of the charge ring at z_0. The linear charge density in any annular ring and at any given displacement plane transverse to the direction of the beam is written as

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$$d\sigma(z,r,t) = -2\pi\rho(z,r,t)r\ dr$$

$$= -2\pi\rho_0(z_0,r_0,t_0) \sum_{NR} \left| r_0\ dr_0\ \frac{dz_0}{dz} \right| \tag{19}$$

by equation (18).

Furthermore, by expressing $\left|\frac{dz_0}{dz}\right|$ as

$$\left|\frac{dz_0}{dz}\right| = \frac{u_z(z_0,r_0,r_0)}{u_z(z,r,t)} \left|\frac{dt_0}{dt}\right|$$

$$= \frac{u_z(z_0,r_0,t_0)}{u_z(r,r,t)} \left|\frac{d\theta_0}{d\theta}\right|$$

we can write equation (19) as

$$d\sigma(z,r,t) = -2\pi\rho_0(z_0,r_0,t_0)\ \frac{u_z(z_0,r_0,t_0)}{u_z(z,r,t)} \sum_{NR} \left|\frac{d\theta_0}{d\theta}\right| r_0\ dr_0 \tag{20}$$

In the computation of bunching current in the second cavity, it is assumed that there is no net current in the radial direction (i.e., small signal theory is assumed); then the incremental current through the annular ring can be written as

$$i(z,t,t) = -2\pi\rho(z_0,r_0,t_0)u_z(z_0,r_0,t_0) \int_0^b r_0\ dr_0 \sum_{NR} \left|\frac{d\theta_0}{d\theta}\right|$$

$$= \frac{2I_0}{b^2} \int_0^b \sum_{NR} \left|\frac{d\theta_0}{d\theta}\right| r_0\ dr_0 \tag{21}$$

In terms of the normalized parameters ξ and ρ, equation (21) can be written as

$$i(\xi,\rho,\theta) = 2I_0\left(\frac{a}{b}\right)^2 \int_0^{b/a} \sum_{NR} \left|\frac{d\theta_0}{d\theta}\right| \rho_0\ d\rho_0 \tag{22}$$

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It is to be noted here that $\rho_0 = r_0/a$ is not the dc beam charge density used prior to equation (22).

The total electron convection current may be conveniently expressed in terms of a Fourier series written in the variables θ as

$$i_n(\xi,\rho,\theta) = A_0 + \sum_{n=1}^{\infty} (A_n \cos n\theta + B_n \sin n\theta) \tag{23}$$

where

$$A_0 = \frac{1}{2\pi} \int_0^{2\pi} i_n(\xi,\rho,\theta)d\theta$$

$$= \frac{2I_0}{2\pi} \left(\frac{a}{b}\right)^2 \int_0^{2\pi} \int_0^{b/a} \rho_0 \, d\rho_0 \, d\theta_0 = I_0 \tag{24}$$

$$A_n = \frac{1}{\pi} \int_0^{2\pi} i_n(\xi,\rho,\theta)\cos n\theta \, d\theta$$

$$= \frac{2I_0}{\pi} \left(\frac{a}{b}\right)^2 \int_0^{2\pi} \int_0^{b/a} \rho_0 \cos n\theta \, d\rho_0 \, d\theta_0$$

$$= \frac{I_0}{\pi} \int_0^{2\pi} \cos n\theta \, d\theta_0 \tag{25}$$

and

$$B_n = \frac{1}{\pi} \int_0^{2\pi} i_n(\xi,\rho,\theta)\sin n\theta \, d\theta$$

$$= \frac{I_0}{\pi} \int_0^{2\pi} \sin n\theta \, d\theta_0 \tag{26}$$

in which θ is the phase of the electron at any point along the stream. This is to be related to the injection phase of the electron θ_0 at the time of entering into the interaction gap space of the first cavity. The interaction gap space is defined (fig. 3) as extending from $z = -(\ell + 2a)$

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to $z = +(\ell + 2a)$, where $2a$ is the tunnel diameter and 2ℓ is the cavity physical gap length. Thus

$$t_2 = t_0 + \frac{2(\ell + 2a)}{u_0} + \frac{L_1 - (\ell + 2a)}{u_{z1}}$$

or

$$\theta_2 = \theta_0 + \theta_g + \frac{\omega L_1 - (\ell + 2a)}{u_{z1}} \tag{27a}$$

In terms of the normalized parameters this can be written as

$$\theta_2 = \theta_0 + \theta_g + \left(\frac{L}{a} - \frac{\ell}{a} - 2\right) \dot{\xi}_1^{-1} \tag{27b}$$

where u_{z1} is the electron velocity at the plane of exit of the first cavity gap to be either computed or substituted from small signal theory.

The bunching current at the midplane of the second cavity gap ($z = z_2$) can now be obtained by, first, letting $\theta = \theta_2$ and, second, substituting equation (27b) for θ in equations (24) and (25). The result is

$$i_n(\xi, \rho, \theta_2) = I_0 + \sum_{n=1}^{\infty} \frac{I_0}{\pi} \cos n\theta \int_0^{2\pi} \cos n\left[\theta_0 + \theta_g + \left(\frac{L}{a} - \frac{\ell}{a} - 2\right) \dot{\xi}_1^{-1}\right] d\theta_0$$

$$+ \sum_{n=1}^{\infty} \frac{I_0}{\pi} \sin n\theta \int_0^{2\pi} \sin n\left[\theta_0 + \theta_g + \left(\frac{L}{a} - \frac{\ell}{a} - 2\right) \dot{\xi}_1^{-1}\right] d\theta_0 \tag{28}$$

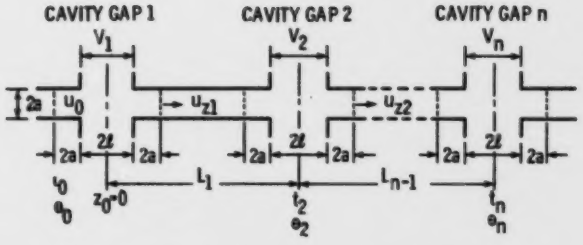

Figure 3. - Geometrical parameters of multicavity klystron.

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The ratio of harmonic current amplitudes to direct current is given by

$$\left|\frac{i_n(\xi,\rho,\theta_2)}{I_0}\right| = \sqrt{A_n^2 + B_n^2} = \frac{1}{\pi^2}\left(\left\{\int_0^{2\pi} \cos n\left[\theta_0 + \theta_g + \left(\frac{L}{a} - \frac{\ell}{a} - 2\right)\xi_1^{-1}\right] d\theta_0\right\}^2\right.$$

$$\left. + \left\{\int_0^{2\pi} \sin n\left[\theta_0 + \theta_g + \left(\frac{L}{a} - \frac{\ell}{a} - 2\right)\xi_1^{-1}\right] d\theta_0\right\}^2\right)^{1/2} \tag{29}$$

for $n = 1, 2, 3, \ldots$. The phase angles of the n^{th} harmonic with reference to a pure cosine or pure sine wave of the same frequency are given by

$$\Delta_n = \tan^{-1}\left(\frac{B_n}{A_n}\right) \tag{30}$$

Once the electron convection current is obtained, the induced current, and hence the induced voltage, can be computed. Since α_2 is small in the second cavity, the current induced there as a consequence of the bunched electron beam traversing the cavity gap can be computed by the following relation:

$$I_{(ind)2} = M_r M_z i_2(\xi,\rho,\theta_2) \tag{31}$$

where $i_2(\xi,\rho,\theta_2)$ is the electron convection current passing through the midplane of the second cavity, and M_r and M_z are radial and axial beam coupling coefficients, respectively, given by the following relations (ref. 4):

$$M_r = \frac{\sqrt{I_0^2(\gamma_e b) - I_1^2(\gamma_e b)}}{I_0(\gamma_e a)} \tag{31a}$$

and

$$M_z = \frac{\sin \beta_e 2\ell}{\beta_e 2\ell} \tag{31b}$$

where $\gamma_e = \sqrt{\beta_e^2 - k_0^2}$ is the radial propagation constant, $k_0 = \omega/c$,

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$\beta_e = \omega/u_0$, and I_0 and I_1 are the modified Bessel functions of the zero and first order, respectively. We note that the axial beam coupling coefficient is to be evaluated at the edge of the cavity gap. The voltage across the second cavity gap (i.e., across the capacitor of the equivalent shunt circuit, fig. 4) is given by

$$V_{g2} = Z_2 I_{(ind)2}$$

where Z_2 is the dynamic shunt impedance of the cavity given by

$$Z_2(\omega) = \frac{R_{sh}}{1 + 2j\delta Q_u}$$

in which R_{sh} is the shunt resistance at resonance, and δ is the fractional deviation from resonance and is defined by

$$\delta = \frac{\omega - \omega_0}{\omega}$$

Thus

$$\alpha_2 = \frac{V_{g2}}{V_0}$$

$$= \frac{M_{r2} M_{2z} i_2(\xi, \rho, \theta_2)}{V_0} \frac{R_{sh2}}{1 + 2j\delta Q_{u2}} \tag{32}$$

where the subscripts 2 imply that these quantities are to be evaluated in cavity gap 2.

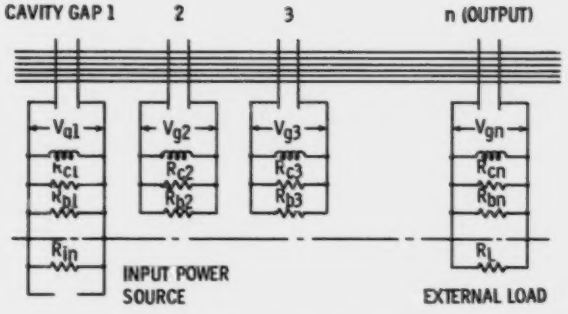

Figure 4. - Schematic of multicavity klystron.

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LARGE SIGNAL ENERGY EXCHANGE AND CALCULATION OF INDUCED CURRENT

In a multicavity high-power klystron, interaction between the bunched electron stream and the cavity gap fields in the third and succeeding cavities up to the output cavity becomes complicated. The simpler method of computing the induced current used in the section CALCULATION OF BUNCHING CURRENT AND VELOCITY MODULATION IN SECOND CAVITY is no longer valid. In this section, a general method of approach is used. This method is based on Shockley (ref. 5) and Ramo (ref. 6) theory.

Consider an electron of charge $-e$ that travels a distance $\vec{dl}$ inside the cavity interaction gap space (fig. 3). The gain or loss of its kinetic energy is given by

$$dW = -e\,\vec{E}_{cct} \cdot \vec{dl} \tag{33}$$

By the theory of Shockley and Ramo, the exchange of energy is related to the induced current that flows in the external circuit (i.e., the cavity) and the induced voltage that appears across the cavity gap by the following relation:

$$dW = -I_{ind} V_{ind}\, dt \tag{34}$$

Setting equation (33) equal to (34) yields

$$I_{ind} = e\,\frac{\vec{E}_{cct}}{V_{ind}} \cdot \frac{\vec{dl}}{dt} = e\,\frac{\vec{E}_{cct}}{V_{ind}} \cdot \vec{u} \tag{35}$$

where $\vec{E}_{cct}$ is the cavity gap field (the circuit field) and $\vec{u} = \vec{dl}/dt$ is the electron velocity.

Since many electrons are present inside the cavity interaction gap space at any instant of time and the electron transit time within the interaction gap space is finite, each elementary current carried at a given moment of time by each elementary charge within the interaction gap space will contribute to the total induced current. These elementary currents in the plane over which the beam enters the interaction gap space, and in the plane located some distance away in the direction in which the beam is traveling, differ not only in velocity but also in phase. By summing up the elementary current or charge within the interaction gap space, we obtain the total induced current

$$I_{ind}(t) = \int_{\tau} \rho\vec{u} \cdot \frac{\vec{E}_{cct}}{V_{ind}}\, d\tau$$

where $\rho(r, z, \theta, \tau)$ is the charge density inside the interaction gap space. Alternatively, in terms of convection current density $\vec{J}_c$ $(= \rho\vec{u})$, equation (37) can be written as

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$$I_{ind}(t) = \int_{\tau} \vec{J}_c \cdot \frac{\vec{E}_{cct}}{V_{ind}} \, d\tau \tag{36}$$

where $d\tau = r\, dr\, d\varphi\, dz$ is the volume element. The integration is taken over the whole volume of space of the interaction gap space occupied by the bunched electrons at the time t. We note that, since the cavity gap field E_{cct} is proportional to the induced voltage V_{ind}, for a given electron velocity u, the induced current is independent of the voltage across the cavity gap.

Using the first expression, equation (36), we find

$$I_{ind}(t) = 2\pi \int_{-(\ell+2a)}^{(\ell+2a)} \int_{0}^{b'(z)} \rho(r,z,t)\vec{u} \cdot \frac{\vec{E}_{cct}}{V_{ind}} \, r\, dr\, dz \tag{37}$$

where $b'(z)$ is the edge of the beam as a function of z and is defined by

$$b'(z) = r_0 + \int_{0}^{t} \dot{r}\, dt \tag{38}$$

in which r_0 is the radial position of the electron at the entrance to the interaction gap space, here to be referred to as the entrance plane of the first cavity gap space.

The integral of equation (37) can be evaluated by applying the charge conservation principle (see fig. 2) to write the charge density $\rho(r,z,t)$ as function of the phase of the injected electrons at the entrance to the interaction gap space of the first cavity as follows:

$$\rho(r,z,t) = \rho_0(r_0,z_0,t_0) \sum \left| \frac{r_0\, dr_0}{r\, dr} \right| \left| \frac{dz_0}{dz} \right|$$

$$= \rho_0(r_0,z_0,t_0) \frac{\int_0^b r_0\, dr_0}{\int_0^{b'(z)} r\, dr} \sum \left| \frac{u_z(r_0,z_0,t_0)dt_0}{u_z(r,z,t)dt} \right|$$

$$= \rho_0(r_0,z_0,t_0) \left[\frac{b_{eq}}{b'(z)} \right]^2 \frac{u_z(r_0,z_0,t_0)}{u_z(r,z,t)} \sum \left| \frac{d\theta_0}{d\theta} \right| \tag{39}$$

where b_{eq} is the equilibrium, or the average, radius of the outer charge ring centroid given by the relation

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$$b_{eq} = \sqrt{2}\,\frac{\omega_p}{\omega_c} = \frac{\sqrt{2}}{R_u^{1/2}\left(1 - \beta^2\right)^{1/2}}\,\frac{\omega_{p0}}{\omega_{c0}}$$

in which ω_{p0} and ω_{c0} are the nonrelativistic plasma and cyclotron angular frequencies, respectively, and R_u is the relativistic reduction factor as given in the section FORMULATION OF RELATIVISTIC EQUATION OF MOTION.

Substituting equation (39) into equation (37), we obtain

$$I_{ind}(t) = 2\pi\rho_0(r_0,z_0,t_0)u_0\left[\frac{b_{eq}}{b'(z)}\right]^2\int_{-(\ell+2a)}^{(\ell+2a)}\int_0^{b'(z)}\frac{\vec{u}}{u_z(r,z,t)}\cdot\frac{\vec{E}_{cct}}{V_{ind}}$$

$$\times\sum\left|\frac{d\theta_0}{d\theta}\right| r\,dr\,dz$$

$$= 2\pi I_0\left[\frac{b_{eq}}{b'(z)}\right]^2\int_{-(\ell+2a)}^{(\ell+2a)}\int_0^{b'(z)}\left(\frac{u_r}{u_z}\frac{E_{r\text{-}cct}}{V_{ind}} + \frac{E_{z\text{-}cct}}{V_{ind}}\right)\sum\left|\frac{d\theta_0}{d\theta}\right| r\,dr\,dz \qquad (40)$$

where $u_z(r_0,z_0,t_0) = u_0$ is the dc beam velocity. In terms of the normalized parameters ξ and ρ, this is written as

$$I_{ind}(t) = 2\pi I_0\left[\frac{b_{eq}}{b'(z)}\right]^2\int_{-\frac{(\ell+2a)}{a}}^{\frac{(\ell+2a)}{a}}\int_0^{\frac{b'(z)}{a}}\left(\frac{\dot{\rho}}{\xi}\frac{E_{r\text{-}cct}}{V_{ind}} + \frac{E_{z\text{-}cct}}{V_{ind}}\right)$$

$$\times\sum\left|\frac{d\theta_0}{d\theta}\right| a^3\rho\,d\rho\,dz \qquad (41)$$

Furthermore the induced current I_{ind} may conveniently be represented by a Fourier series in the variable θ as follows:

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$$I_{ind(n)}(\omega,t) = \frac{1}{2\pi}\int_0^{2\pi} I_{ind}(t)d\theta + \sum_{n=1}^{\infty}\frac{\cos n\theta}{\pi}\int_0^{2\pi} I_{ind}(t)\cos n\theta\, d\theta$$

$$+ \sum_{n=1}^{\infty}\frac{\sin n\theta}{\pi}\int_0^{2\pi} I_{ind}\sin n\theta\, d\theta \qquad (42)$$

where $I_{ind}(t)$ is given by equation (41).

The two integrals in equation (42) can be evaluated, first, by transforming the variable θ in terms of θ_0, the electron injection phase at the entrance plane to the interaction gap space of the first cavity. With reference to figure 3, we note that the times, t_1, t_2, t_3, . . ., t_n, taken by electrons to pass through the rf gaps are interrelated:

$$t_3 = t_2 + \frac{\ell + 2a}{u_{z1}} + \frac{L_2 - (\ell + 2a)}{u_{z2}}$$

$$= t_0 + \frac{2(\ell + 2a)}{u_0} + \frac{L_1}{u_{z1}} + \frac{L_2 - (\ell + 2a)}{u_{z2}}$$

In terms of the normalized parameter ξ, this can be written as

$$\theta_3 = \theta_0 + \theta_g + \frac{L_1}{a}\dot{\xi}_1^{-1} + \left(\frac{L_2}{a} - \frac{\ell}{a} - 2\right)\dot{\xi}_2^{-1}$$

In general

$$\theta_n = \theta_0 + \theta_g + \sum_{n=1}^{n=2}\frac{L_{n-2}}{a}\dot{\xi}_1^{-1} + \left(\frac{L_{n-1}}{a} - \frac{\ell}{a} - 2\right)\dot{\xi}_2^{-1} \qquad (43)$$

where $\theta_g = 2(\ell + 2a)(\omega/u_0)$ is the dc transit angle of the electron in traversing the first interaction gap space.

Next, by relating the cavity gap field $\vec{E}_{cct}$ with the induced voltage V_{ind} through equations (11) to (13), we can write

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$$\left|\frac{\vec{E}_{cct}}{V_{ind}}\right| = \frac{mV_n \cos(\theta_n + \Delta_n)}{2 \sinh(m\ell)} \frac{1}{V_n \cos(\theta_n + \Delta_n)} \begin{pmatrix} G \\ F \end{pmatrix}$$

$$= \frac{m}{2 \sinh(m\ell)} \begin{pmatrix} G \\ F \end{pmatrix}$$

Furthermore, for simplicity, equation (42) is written in terms of the Fourier coefficients, A_0, A_n, and B_n. By a substitution of equation (43) into equation (42), we obtain

$$A_0 = \frac{mI_0 a^3}{2 \sinh(m\ell)} \left[\frac{b_{eq}}{b'(z)}\right]^2 \int_0^{2\pi} \int_{-\frac{(\ell+2a)}{a}}^{\frac{(\ell+a)}{a}} \int_0^{\frac{b'(z)}{a}} X \rho \, d\rho \, d\xi \, d\theta_0 \qquad (44)$$

$$A_n = \frac{mI_0 a^3}{2 \sinh(m\ell)} \left[\frac{b_{eq}}{b'(z)}\right]^2 \int_0^{2\pi} \int_{-\frac{(\ell+2a)}{a}}^{\frac{(\ell+a)}{a}} \int_0^{\frac{b'(z)}{a}} X \cos \rho \, d\rho \, d\xi \, d\theta_0 \qquad (45)$$

$$B_n = \frac{mI_0 a^3}{2 \sinh(m\ell)} \left[\frac{b_{eq}}{b'(z)}\right]^2 \int_0^{2\pi} \int_{-\frac{(\ell+2a)}{a}}^{\frac{(\ell+a)}{a}} \int_0^{\frac{b'(z)}{a}} X \sin \rho \, d\rho \, d\xi \, d\theta_0 \qquad (46)$$

where

$$X = \frac{\dot{\rho}}{\dot{\xi}} \begin{pmatrix} G_\rho \\ F_\rho \end{pmatrix} + \begin{pmatrix} G_\xi \\ F_\xi \end{pmatrix} \qquad (47)$$

The G's are given by equations (11) to (12), and the variable θ is given by equation (43). The amplitudes of the harmonic to dc can be obtained by using the relation

$$\left|\frac{I_{(ind)n}}{I_0}\right| = \sqrt{A_n^2 + B_n^2} \qquad (48)$$

and the phase angle is related by

$$\Delta_n = \tan^{-1} \frac{B_n}{A_n} \qquad (49)$$

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With the induced current obtained from equations (42) to (49), the modulation index for the third and succeeding cavities up to the penultimate cavity can be computed through an iteration procedure. To illustrate this method of approach, let us consider a case of computing α_3 (i.e., the gap voltage of the third cavity) as an example:

(1) Find the induced current as computed at the midplane of the third cavity gap, called $I_{(ind)3}$.

(2) Let the gap voltage $V_{g3} = 0$, and compute the gap voltage using the relation

$$V_{g3a} = Z_3(\omega) I_{(ind)3}$$

where

$$Z_3(\omega) = \frac{R_{sn}}{1 + 2j\delta Q_{u3}}$$

(3) Using the value of V_{g3a} as obtained in step (2), formulate and solve the equations of motion, equations (16) and (17), and thus obtain the induced current, called i_{3a}.

(4) Using i_{3a}, as in step (2), compute the gap voltage, called the gap voltage V_{g3b}.

(5) Repeat steps (3) and (4) until a gap voltage V_{g3k} is obtained so that the following condition converges:

$$\frac{V_{g3k}}{V_0} = \alpha_3$$

In the case of the output cavity, the gap voltage V_{g0} is simply

$$V_{g0} = Z_{out}(\omega) I_{(ind)0} \tag{50}$$

where $I_{(ind)0}$ is the current induced in the output cavity, and $Z_{out}(\omega)$ is the dynamic shunt impedance of the output cavity given by

$$Z_{out}(\omega) = \frac{R_{sh0}}{1 + j2\delta Q_{u0}} \tag{51}$$

Power delivered to the external load of the cavity is obtained by

$$P_{out} = \frac{1}{2} \mathrm{Re} I_{(ind)0} V^*_{g0} \tag{52}$$

where V^*_{g0} is the complex conjugate of V_{g0}. Finally, the output cavity efficiency can be computed in the usual way:

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$$\eta = \frac{P_{out}}{P_{in}} = \frac{(1/2)\mathrm{Re}I_{(ind)0}V^{*}_{g0}}{V_0 I_0} \tag{53}$$

where V_0 is the dc beam voltage, and I_0 is the dc beam current, which is related to the perveance by the relation

$$K_r = K_0\left(1 - \frac{3}{28}\frac{V_0}{V_e}\right) \tag{54}$$

where $K_0 = I_0/V_0^{3/2}$ is the nonrelativistic perveance, and $V_e = m_0c^2/e = 5.110\times10^5$ volts is the equivalent beam voltage.

SUMMARY

Three-dimensional equations of motion of relativistic electrons in an axisymmetric field in the presence of space-charge forces has been obtained. The field-electron interactions in various interaction gaps and in various drift spaces are investigated by using a Lagrangian formulation of the hydrodynamic model. The equations of motion are used to find the velocity of each disk of electrons, its phase with respect to the interaction gap voltage, and its radius at a specified position in various interaction gaps and in various drift spaces. The velocity modulation and electron bunching in the input and second cavities are formulated by using the small signal approach; however, both the radial and axial coupling coefficients are used to compute current. The induced current in the third and succeeding cavities up to and including the output cavity is computed on the basis of Shockley and Ramo theory. Power and efficiency are calculated in the usual way.

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APPENDIX A

MODIFICATION OF PARAMETERS DUE TO RELATIVISTIC VELOCITY

Mass Increase

When the electrons are accelerated through very high potentials, they experience an increase in mass which modifies the velocity-potential relationship. The increase of the electronic mass with increasing electron velocity is given from the transformation law originally proposed by H. A. Lorentz (1904):

$$m = \frac{m_0}{\sqrt{1 - \beta^2}} \tag{A1}$$

where $\beta \triangleq u/c$, as usual in relativistic formulas; $c = 3 \times 10^8$ meters per second is the velocity of light; m is the mass of the electron at the velocity u; and m_0 is the electronic rest mass at zero velocity.

Velocity of Electron

If W is the total energy of the electron in joules, the relation

$$W = mc^2 \tag{A2}$$

expresses the equivalence of the instantaneous mass and the inertia of energy. The total energy of an electron is the sum of the energy inherent in its mass plus its kinetic energy. The total energy change of an electron when it is accelerated through a potential of V_0 volts, can, according to equation (A-2), be expressed by

$$eV_0 = \Delta W = c^2(m - m_0)$$

$$= m_0c^2 \left(\frac{m}{m_0} - 1\right) \tag{A3}$$

Thus

$$\frac{m}{m_0} = \frac{e}{m_0c^2} V_0 + 1 \tag{A4}$$

By introducing the energy equivalent of the electron rest mass m_0 as

$$eV_e = m_0c^2 \tag{A5}$$

we obtain

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$$V_e = \frac{m_0 c^2}{e} = 0.5110 \times 10^6 \text{ volts} \tag{A6}$$

Hence equation (A4) can be written as

$$\frac{m}{m_0} = \frac{V_0}{V_e} + 1 \tag{A7}$$

When this is substituted into equation (A1), we obtain the relation between the electron velocity u and the accelerating potential V:

$$u_0 = c\left[1 - \left(1 + \frac{V_0}{V_e}\right)^{-2}\right]^{1/2} \tag{A8a}$$

or, alternatively,

$$u_0 = \sqrt{2\frac{e}{m_0}V}\,\frac{\sqrt{1 + \frac{1}{2}\frac{V_0}{V_e}}}{1 + \frac{V_0}{V_e}} = c\,\frac{\sqrt{V(V + 1022)}}{V + 511}$$

$$= u_{00}R_u \text{ (for } V \text{ in kilovolts)} \tag{A8b}$$

where

$$u_{00} = \sqrt{2\frac{e}{m_0}V_0}$$

is the nonrelativistic expression of the electron velocity, and

$$R_u = \frac{\sqrt{1 + \frac{1}{2}\frac{V_0}{V_e}}}{1 + \frac{V}{V_e}}$$

is the relativistic correction factor.

Angular and Plasma Frequency of Electron Stream

The angular frequency of revolution of the electron in the homogeneous magnetic field B is defined as the cyclotron frequency given by

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$$\omega_c = \frac{e}{m} B \tag{A9}$$

At moderate electron velocities, the increase in electronic mass is negligibly small; hence ω_c is considered to be independent of the velocity of the electron. However, this is not true for the high-velocity electron. The cyclotron frequency for large velocities follows by substitution of the relativistic mass of equation (A1) for m:

$$\omega_c = \frac{e}{m_0} B\left[1 - \left(\frac{u}{c}\right)^2\right]^{1/2}$$

$$= \omega_{c0}\left[1 - \left(\frac{u}{c}\right)^2\right]^{1/2} \tag{A10}$$

where $\omega_{c0} \triangleq (e/m_0)B$ is the nonrelativistic cyclotron frequency. In terms of V_e, equation (A10) can be written as

$$\omega_c = \frac{Bc^2}{V_0 + V_e} \tag{A11}$$

In like manner, the relativistic plasma frequency can be found by substituting the relativistic mass, equation (A1), and the relativistic velocity, equation (A8b), for m and u, respectively:

$$\omega_p = \sqrt{\frac{\frac{e}{m} I_0}{\epsilon_0 \pi u_0}}$$

$$= \omega_{p0}\left(1 + \frac{1}{2}\frac{V}{V_e}\right)^{-1/4}$$

where $\omega_{p0} = \sqrt{\frac{\frac{e}{m_0} I_0}{\epsilon_0 \pi u_{00}}}$ is the nonrelativistic plasma frequency, and u_{00}, as given in equation (A8b), is the nonrelativistic velocity of the electron.

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APPENDIX B

ENERGY CONSERVATION PRINCIPLE

Consider an electron which has been accelerated through a potential V_0 volts and gained kinetic energy in the amount of eV_0; then, by the equivalence of mass and energy principle, the change in the total energy of the charged particle is

$$\delta W = eV_0 = c^2(m - m_0) = c^2 \delta m \tag{B1}$$

by equation (A3) of appendix A.

Work done by the force in moving the charged particle of mass m a distance $\vec{dr}$ is given by

$$dW = \vec{F} \cdot \vec{dr} = e\left(\vec{E} + \vec{u} \times \vec{B}\right) \cdot \vec{dr} \tag{B2}$$

where the Lorentz force law has been involved on the right side of the equation. Now, power is the rate of change of energy; thus, dividing both sides of equation (B2) by dt, we obtain

$$\frac{dW}{dt} = e\left(\vec{E} + \vec{u} \times \vec{B}\right) \cdot \frac{\vec{dr}}{dt}$$

$$= e\left(\vec{E} \cdot \vec{u}\right) \tag{B3}$$

where $\vec{u} = \vec{dr}/dt$, and $\left(\vec{u} \times \vec{B}\right) \cdot \vec{u}$ is identically equal to zero. Equation (B1) is differentiated with respect to time, and the resultant expression is used to write equation (B3) as follows:

$$\frac{dm}{dt} = \frac{e}{c^2}\left(\vec{E} \cdot \vec{u}\right)$$

or

$$\frac{d}{dt}\left(\frac{m}{m_0}\right) = -\frac{e}{m_0 c^2}\left(\vec{E} \cdot \vec{u}\right) \tag{B4a}$$

$$\frac{d}{dt}\left(\frac{m}{m_0}\right) = -\frac{\vec{E} \cdot \vec{u}}{V_e} \tag{B4b}$$

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In a cylindrical coordinate system, this is written as

$$\frac{d}{dt}\left(\frac{m}{m_0}\right) = -\frac{\dot{r}E_r + r\dot{\varphi}E_\varphi + \dot{z}E_z}{V_e} \qquad \text{(B4c)}$$

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REFERENCES

1. Kosmahl, Henry G.: A Novel, Axisymmetric, Electrostatic Collector for Linear Beam Microwave Tubes. NASA TN D-6093, 1971.

2. Kosmahl, Henry G.; and Branch, Garland M., Jr.: Generalized Representation of Electric Fields in Interaction Gaps of Klystrons and Traveling-Wave Tubes. IEEE Trans. Electron Devices, ED-20, no. 7, July 1973, pp. 621-629.

3. Kosmahl, Henry G.; and Albers, Lynn U.: Three-Dimensional Evaluation of Energy Extraction in Output Cavities of Klystron Amplifiers. IEEE Trans. Electron Devices, vol. ED-20, no. 10, Oct. 1973, pp. 883-890.

4. Branch, Garland M., Jr.: Electron Beam Coupling in Interaction Gaps of Cylindrical Symmetry. IRE Trans. Electron Devices, vol. ED-8, no. 3, May 1961, pp. 193-207.

5. Shockley, W.: Currents to Conductors Induced by a Moving Point Charge. J. Appl. Phys., vol. 9, Oct. 1938, pp. 635-636.

6. Ramo, S.: Currents Induced by Electron Motion, Proc. IRE, vol. 27, no. 9, Sept. 1939, pp. 584-585.

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16. Abstract

This report describes a theoretical investigation of three-dimensional relativistic klystron interaction. The relativistic axisymmetric equations of motion are derived from the time-dependent Lagrangian function for a charged particle in electromagnetic fields. An analytical expression of the fringing RF electric and magnetic fields within and in the vicinity of the interaction gap and the space-charge forces between axially and radially elastic deformable rings of charges are both included in the formulation. This makes an accurate computation of electron motion through the tunnel of the cavities and the drift tube spaces possible. Method of analysis is based on Lagrangian formulation. Bunching is computed using a disk model of electron stream in which the electron stream is divided into axisymmetric disks of equal charge and each disk is assumed to consist of a number of concentric rings of equal charges. The individual representative groups of electrons are followed through the interaction gaps and drift tube spaces. Induced currents and voltages in interacting cavities are calculated by invoking the Shockley-Ramo theorem.

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